CANDIDATE'S NUMBER

(For the use of the Department only)

Test

Math. 30
Aug. 1968

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DEPARTMENT
EDUCATION ALBERTA

HIGH SCHOOL AND UNIVERSITY MATRICULATION
EXAMINATIONS BOARD

DEPARTMENTAL EXAMINATIONS, 1968
AUGUST

MATHEMATICS 30 (NEW)

Time — 2 hours.

Note — The total time allowed for this paper is 2 hours. Distribute this time to best advantage by reading the paper, answering the questions, and reviewing the work already done. DO NOT spend too much time on any one question.

Total Possible Mark — 110.

Knott's Mathematical Tables may be used and these tables will be supplied by the Presiding Examiner. Candidates are NOT permitted to use slide rules.

SECTION A

Section A of the paper contains questions 1 - 30 inclusive. Answers to these questions are to be recorded on the separate ANSWER SHEET which will be given to you by the Presiding Examiner. Each question has four suggested answers of which only one is correct. Select the correct answer in each case and record your choice on the separate ANSWER SHEET provided as shown in the example below. There is no penalty for incorrect answers.

EXAMPLE	ANSWERS	ANSWER SHEET
The product of 8 and 12 is	A. 20 B. 96 C. 48 D. 72	A ☐ B ■ C ☐ D ☐ E ☐

You are to use ORDINARY HB PENCIL. Make certain your answer marks are heavy and black. If you change your mind about an answer, be sure to completely erase your first mark. There should be only one answer marked for each question. The ANSWER SHEET will be machine scored; consequently, candidates should insure that no stray pencil marks appear on the ANSWER SHEET. Write your name and school in the space provided on the ANSWER SHEET; but DO NOT write your name on this booklet.

SECTION B

Section B of this paper consists of questions 31 - 43 inclusive. Complete solutions for each question should be shown in the spaces provided in the booklet. Some marks may be obtained for partial solutions. DO NOT put work which is to be read by the sub-examiner in the ROUGH WORK ONLY column. Finished work in this section may be done in either pencil or ink (blue or black).

DO NOT FOLD EITHER THE ANSWER SHEET OR THE TEST BOOKLET.

Place your answer sheet **BEHIND** the test booklet and **INSERT BOTH IN THE SAME ENVELOPE** in such a way that the **UPPER LEFT-HAND CORNER OF THE BOOKLET** (Candidate's number) shows through the **SLASHED CORNER** of the answer envelope.

BE SURE TO SEAL THE ENVELOPE.

Machine Score

Book Score

TOTAL

(For the use of Sub-Examiners only)

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3054
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CURR HIST

Questions 1 - 30 inclusive have a value of two marks each. In each case select the CORRECT ANSWER and record your choice on the separate ANSWER SHEET that has been provided.

1. The truth values of the statement $P \rightarrow Q$ in the table should read:

P	Q	$P \rightarrow Q$
T	T	?
T	F	?
F	T	?
F	F	?

A. T, T, T, F
B. T. F, T, T
C. T, F, F, F
D. T, F, F, T

2. Express with a rational denominator and simplify: $(\sqrt{3} + \sqrt{2})^{-2}$

A. $5 - 2\sqrt{6}$
B. $2\sqrt{6} - 5$
C. $5 + 2\sqrt{6}$
D. $\frac{1}{5}$

3. If $f(x) = 3x + \frac{1}{x^2}$ and $g(x) = 2x - 1$, evaluate $f\left[\, g\left(\frac{2}{3}\right) \right]$

A. 10
B. $\frac{10}{9}$
C. $\frac{17}{4}$
D. $\frac{15}{2}$

4. If $f : x \rightarrow x^3$, $x \in R$, the graph is symmetric with respect to

A. $x = 3$
B. y–axis
C. x–axis
D. origin

5. Select the appropriate quantifiers to convert the following sentence into a true statement:

$a(b + c) = ab$; a, b, c are elements of R.

A. $\forall a,\ \forall b,\ \exists c,\ a,\ b,\ c \in R,\ a(b + c) = ab$
B. $\exists a,\ \exists c,\ \forall c,\ a,\ b,\ c \in R,\ a(b + c) = ab$
C. $\exists a,\ \forall b,\ \exists c,\ a,\ b,\ c \in R,\ a(b + c) = ab$
D. $\forall a,\ \exists b,\ \exists c,\ a,\ b,\ c \in R,\ a(b + c) = ab$

FOR ROUGH WORK

(No marks given for work in this space)

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6. The inverse function f^{-1} of $f: x \rightarrow \log_5 x,\ x > 0,\ x \in R$ is

A. $f^{-1}: x \rightarrow \log_x 5,\ x > 0,\ x \in R$
B. $f^{-1}: x \rightarrow x^5,\ x \in R$
C. $f^{-1}: x \rightarrow 5^x,\ x \in R$
D. $f^{-1}: y \rightarrow \log_5 y,\ y \in R$

7. The function f, g, whose graph is a straight line with slope 0 and y-intercept of -2 is

A. $g : x \rightarrow -2x,\ x \in R$
B. $g : x \rightarrow -2,\ x \in R$
C. $g : x \rightarrow x - 2,\ x \in R$
D. $g : x \rightarrow 0,\ x \in R$

8. The graph of the function $y = -2x^2 + 4x - 1$

A. lies entirely above the x-axis
B. lies entirely below the x-axis
C. intersects the x-axis in two distinct points
D. is tangent to the x-axis

9. If m and n are roots of the equation $2x^2 + 10x - 3 = 0$, evaluate $m^2 + n^2$

A. 22
B. -28
C. -22
D. 28

10. If f is a geometric sequence such that $f_8 = 3$ and $f_{13} = 96$, the common ratio is

A. 32
B. $\frac{93}{5}$
⑭
C. $\frac{32}{5}$
D. 2

11. Find the sum of the series $\left(\frac{1}{10}\right)^0 + \left(\frac{1}{10}\right)^1 + \left(\frac{1}{10}\right)^2 + \left(\frac{1}{10}\right)^3$

A. 0.1111
B. 1.111
C. 11.11
D. 0.0111

12. If $\dfrac{a + bi}{2 - i} = 4 + 2i$, the values of a and b are

A. $a = 10,\ b = 0$
B. $a = 6,\ b = 0$
C. $a = 0,\ b = 10$
D. $a = 0,\ b = 6$

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FOR ROUGH WORK

(No marks given for work in this space)

13. The two roots of $x^2 - 6x + 11 = 0, x \in C$, are

A. $3 + i, 3 - i$
B. $3 + i\sqrt{3}, 3 - i\sqrt{3}$
C. $3 + i\sqrt{2}, 3 - i\sqrt{2}$
D. $3 + 2i, 3 - 2i$

14. The distance between the two points $-2 - 4i$ and $-7 + 8i$ in the complex plane is

A. 13
B. 169
C. $5 - 12i$
D. $-5 + 12i$

15. A sum of $150 is deposited at the end of each year in a fund that pays 4% per annum compounded semi-annually. The accumulated amount of the annuity after the 10th deposit has been made, is

A. $\dfrac{1500(1.02^{20} - 1)}{1.02^2 - 1}$
B. $\dfrac{150(1.02^{20} - 1)}{1.02^2 - 1}$
C. $\dfrac{150(1.04^{10} - 1)}{1.04 - 1}$
D. $\dfrac{1500(1.04^{10} - 1)}{1.04 - 1}$

16. Evaluate: $b(2; 6, \frac{1}{4})$

A. $\binom{6}{2}\left(\frac{1}{4}\right)^4\left(\frac{3}{4}\right)^2$
B. $\binom{6}{2}\left[\left(\frac{1}{4}\right)^2 + \left(\frac{3}{4}\right)^4\right]$
C. $\binom{6}{2}\left(\frac{1}{4}\right)^2\left(\frac{3}{4}\right)^4$
D. $\binom{6}{2}\left[\left(\frac{1}{4}\right)^4 + \left(\frac{3}{4}\right)^2\right]$

17. Solve for n : $(n)_3 = 3[(n - 1)_2]$

A. 9
B. 5
C. 6
D. 3

18. If a 5-volume set of books is randomly placed on a shelf, what is the probability that they will be arranged in either proper or reverse order?

A. 120
D. 118
C. $\frac{1}{12}$
D. $\frac{1}{60}$

16

$\frac{1}{120} + \frac{1}{120}$

FOR ROUGH WORK

(No marks given for work in this space)

19. The graph of the relation $\{(x, y) \mid 2x^2 - 5xy = 3y^2,\ x, y \in R\}$ is

A. two intersecting straight lines
B. hyperbola
C. parabola
D. ellipse

20. The length of the transverse axis of the hyperbola defined by the equation $\frac{x^2}{9} - \frac{y^2}{5} = 1$ is

Geo

A. $2\sqrt{5}$
B. 6
C. 3
D. $\sqrt{5}$

21. The standard equation of the circle with center $\left(\frac{1}{3}, -\frac{1}{2}\right)$ and tangent to the y-axis is

Geo

A. $\left(x + \frac{1}{3}\right)^2 + \left(x - \frac{1}{2}\right)^2 = \frac{1}{4}$
B. $\left(x - \frac{1}{3}\right)^2 + \left(y + \frac{1}{2}\right)^2 = \frac{1}{4}$
C. $\left(x - \frac{1}{3}\right)^2 + \left(y + \frac{1}{2}\right)^2 = \frac{1}{9}$
D. $\left(x + \frac{1}{3}\right)^2 + \left(y - \frac{1}{2}\right)^2 = \frac{1}{9}$

22. If a parabola is defined by an equation of the form $y^2 = 4px$, the equation of the parabola with focus a point on the line defined by $5x - 3y - 15 = 0$, is

Geo

A. $y^2 = -20x$
B. $y^2 = 1x$
C. $y^2 = 12x$
D. $y^2 = 20x$

23. The coordinates of the foci of the hyperbola whose defining equation is $9x^2 - 4y^2 = -36$ are

Geo

A. $(0, \sqrt{5}),\ (0, -\sqrt{5})$
B. $(\sqrt{13}, 0),\ (-\sqrt{13}, 0)$
C. $(\sqrt{5}, 0),\ (-\sqrt{5}, 0)$
D. $(0, \sqrt{13}),\ (0, -\sqrt{13})$

24. $A \cup (B \cap C)$ is equivalent to

A. $(A \cup B) \cup (A \cup C)$
B. $(A \cap B) \cup (A \cap C)$
C. $(A \cup B) \cap (A \cup C)$
D. $(A \cap B) \cap (A \cap C)$

FOR ROUGH WORK

(No marks given for work in this space)

25. Using the digits 0, 3, 6, 9 the number of positive odd integers less than 1000, where repetitions are allowed, is

A. 4 !
B. 4 × 4 × 2
⑮ C. 3 !
D. 3 × 4 × 2

26. The total number of 2-arrangements or 4-arrangements of a set A with 7 elements is

A. (7 × 6) + (7 × 6 × 5 × 4)
B. (7 × 6) × (7 × 6 × 5 × 4)
⑮ C. $7^2 + 7^4$
D. $7^2 \times 7^4$

27. Using all the letters of the word ABSCISSA, the number of distinct arrangements is

A. 40,320
⑮ B. 3,600
C. 120
D. 3,360

28. Eleven points lie on the circumference of a circle. The number of inscribed hexagons that may be drawn using these points as vertices is

A. $(11)_6$
⑮ B. $\binom{11}{6}$
C. 6 !
D. $\frac{(11)_6}{6}$

29. The sixth term in the expansion of $(x + 2y)^{12}$ is

A. $\binom{12}{6} x^6 (2y)^6$
⑮ B. $\binom{12}{5} x^7 \cdot 2y^5$
C. $\binom{12}{5} x^7 (2y)^5$
D. $\binom{12}{6} x^6 \cdot 2y^6$

30. There are 4 green, 3 white and 5 red marbles in a bag. The probability that two marbles drawn simultaneously from the bag will both be white, is

A. $\frac{1}{11}$
⑯ B. $\frac{19}{44}$
C. $\frac{1}{24}$
D. $\frac{1}{22}$

FOR ROUGH WORK

(No marks given for work in this space)

Values

SECTION B

Complete solutions for questions 31 - 43 should be shown in the spaces provided in the booklet.

ROUGH WORK ONLY
(No marks given for work in this space)

3 **31.** Determine the domain and range of
$\{(x, y) \mid y^2 = 3x - 9,\ x, y \in R\}$

3 **32.** Prove: $\binom{n}{r} = \binom{n}{n-r}$

Values

3 **33.** Which term in the expansion of $\left(a - \frac{1}{a}\right)^6$ does not contain 'a' ?

ROUGH WORK ONLY
(No marks given for work in this space)

3 **34.** A red and a green die are thrown. Let A be the event that the sum of the resulting numbers is 7 and let B be the event that the red die shows 6. Find $P(A \mid B)$.

Values

3 **35.** If three cards are drawn in succession from a deck of 52 cards, find the probability of getting an ace, a king, and a queen in that order. (Leave your answer in any correct form.)

ROUGH WORK ONLY
(No marks given for work in this space)

3 **36.** Find the probability of throwing a double at least once in 3 throws of 2 dice.

4 **37.** In an arithmetic sequence, $f_4 = 29$ and $f_{10} = 47$. Find the sum of the first 50 terms of the series.

Values

4 **38.** By completing the square of the terms of x and y, determine the radius and coordinates of the center of the circle defined by the equation:

$$9x^2 + 9y^2 - 12x + 6y - 4 = 0$$

ROUGH WORK ONLY
(No marks given for work in this space)

4 **39.** From 4 men and 3 women a committee of 3 persons is to be chosen at random. What is the probability that the committee will contain at least 2 men?

Values

5 **40.** A car stall rental has 50 outlets. When the rent is $10 per month all the outlets are occupied. Each increase of $1 in the rent results in two outlets remaining unoccupied. It costs the owner $3 per month to maintain each occupied outlet. What rent should the owner charge to realize the maximum profit?

ROUGH WORK ONLY
(No marks given for work in this space)

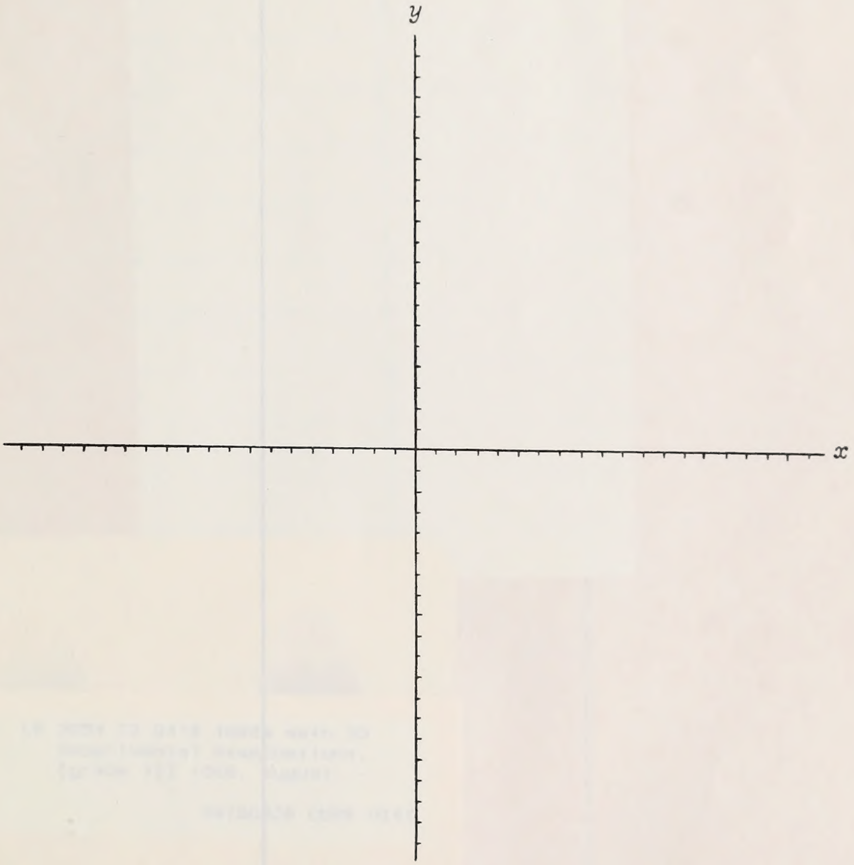
Values

5 **41.** (a) With reference to the same set of axes, sketch the graphs defined by

$$\frac{x^2}{144} + \frac{y^2}{25} = 1 \text{ and } 2x - y - 8 = 0.$$

(b) Shade the region containing all points $P(x, y)$ such that

$$2x - y - 8 \leq 0 \text{ and } \frac{x^2}{144} + \frac{y^2}{25} - 1 \leq 0.$$



FOR ROUGH WORK

(No marks given for work in this space)

Values

5 **42.** $y = mx + 4$ is the equation of a family of lines with y-intercept of 4. Determine the values of m for which the lines intersect the circle $x^2 + y^2 = 2$ at two distinct points.

ROUGH WORK ONLY
(No marks given for work in this space)

Values

5 **43.** Prove by mathematical induction that

$$2 + 6 + 18 + \ldots + 2(3^{n-1}) = 3^n - 1$$

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